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Exact solutions for different coupled nonlinear Maccari's systems

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Abstract. In this paper, we apply the first integral method to study the exact solutions of different coupled nonlinear Maccari's systems.

1 Introduction

Nonlinear evolution equations in mathematical physics play a major role in various fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. The present paper investigates for the first time the applicability and effectiveness of the first integral method on the following *coupled Maccari's systems*.
We consider the coupled Maccari's system:

$$\begin{cases} iQ_t + Q_{xx} + L[Q] = 0, \\ R_t + R_y + M[Q] = 0. \end{cases} \tag{1.1}$$

where $L[Q] = RQ$ and $M[Q] = (|Q|^2)_x$. Also, we can introduce *two coupled nonlinear Maccari's system*:

$$\begin{cases} iQ_t + Q_{xx} + RQ = 0, \\ iS_t + S_{xx} + RS = 0, \\ R_t + R_y + (|Q+S|^2)_x = 0. \end{cases} \tag{1.2}$$

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Moreover, we can introduce *three coupled nonlinear Maccari's system*:

$$\begin{cases} iQ_t + Q_{xx} + RQ = 0, \\ iS_t + S_{xx} + RS = 0, \\ iN_t + N_{xx} + RN = 0, \\ R_t + R_y + (|Q+S+N|^2)_x = 0. \end{cases} \tag{1.3}$$

For the first time, Maccari derived a similar system from the Kadomtsev-Petviashvili equation by using asymptotically exact reduction method based Fourier expansion and spatiotemporal rescaling [13]. Maccari's system is a kind of nonlinear evolution equations that are often presented to describe the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, plasma physics, nonlinear optic, etc. Zhang used Exp-function method for seeking exact solutions of Maccari's system [16].

The rest of this paper is organized as follows. Section 2 is a brief introduction to the first integral method. In section 3, 4 and 5 implementing the first integral method, some exact solutions for some coupled nonlinear Maccari's systems are reported and this describes ability and reliability of the method. A conclusion remark and future directions for research are all summarized in the last section.

2 Review of the first integral method

The first integral method is very popular and we can find a lot of publications with applications of this method in many journals. This method was further developed by the same author in [4, 5, 6, 7, 8, 9, 10] and some other mathematicians [2, 11, 12, 15]. This method was first proposed by Z. Feng in solving Burger-KdV equation [3] which is based on the ring theory of commutative algebra. Consider a general nonlinear PDE in the form

$$P(u, u_t, u_x, u_y, u_{xx}, u_{tt}, u_{yy}, u_{xt}, u_{xy}, u_{yt}, u_{xxx}, ...) = 0. \tag{2.1}$$

Using the wave variable $\eta = x + \beta y - 2kt$ carries (2.1) into the following ODE:

$$Q(U, U', U'', U''', ...) = 0, \tag{2.2}$$

where prime denotes the derivative with respect to the same variable η.

Next, we introduce new independent variables $x = u$, $y = u_\eta$ which change to a system of ODEs

$$\begin{cases} x' = y \\ y' = f(x, y) \end{cases} \tag{2.3}$$

According to the qualitative theory of differential equations [1], if one can find two first integrals to System (2.3) under the same conditions, then analytic solutions to (2.3) can be solved directly. However, in general, it is difficult to realize this even for a single first integral, because for a given plane autonomous system, there is no general theory telling us how to find it's first integrals in a systematic way. A key idea of our approach here to find first integral is to utilize the Division Theorem. For convenience, first let us recall the Division Theorem for two variables in the complex domain $\mathbb{C}$ [4].

Division Theorem. Suppose that $P(x, y)$ and $Q(x, y)$ are polynomials of two variables x and y in $\mathbb{C}[x, y]$ and $P(x, y)$ is irreducible in $\mathbb{C}[x, y]$. If $Q(x, y)$ vanishes at all zero points of $P(x, y)$, then there exists a polynomial $G(x, y)$ in $\mathbb{C}[x, y]$ such that

$$Q(x,y) = P(x,y)G(x,y).$$

In the following, we obtain the exact solution for (1.1)-(1.3).

3 Exact solutions for Eq. (1.1)

In order to seek exact solutions of system (1.1), we suppose

$$Q(x,y,t) = u(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{3.1}$$

Where k,α and λ are constants to be determined later, l is an arbitrary constant. Substituting Eq. (3.1) into system (1.1) and yields

$$\begin{cases} i(u_t+2ku_x)+u_{xx}-(\lambda+k^2)u+uR=0, \\ R_t+R_y+(u^2)_x=0, \end{cases} \tag{3.2}$$

Using the transformation

$$u=u(\eta), \quad R=R(\eta), \quad \eta=x+\beta y-2kt, \tag{3.3}$$

where β is a constant, system (3.2) become the following

$$\begin{cases} u''-(\lambda+k^2)u+uR=0, \\ (\beta-2k)R'+(u^2)'=0, \end{cases} \tag{3.4}$$

Where prime denotes the differential with respect to η. Integrating the second segment of Eq. (3.4) with respect to η and taking the integration constant as zero yields

$$R=\frac{-1}{\beta-2k}u^2. \tag{3.5}$$

Substituting Eq. (3.5) into the first segment of (3.4) yields

$$u''-(\lambda+k^2)u-\frac{1}{\beta-2k}u^3=0. \tag{3.6}$$

Next, we introduce new independent variables $x=u$, $y=u_\eta$ which change Eq. (3.6) to a system of ODEs

$$\begin{cases} x'=y \\ y'=(\lambda+k^2)x+\frac{1}{\beta-2k}x^3. \end{cases} \tag{3.7}$$

Now, we are applying the Division Theorem to seek the first integral to (3.7). Suppose that $x=x(\eta)$ and $y=y(\eta)$ are the nontrivial solutions to (3.7), and

$$p(x,y)=\sum_{i=0}^{m}a_i(x)y^i,$$

is an irreducible polynomial in $\mathbb{C}[x,y]$ such that

$$p(x(\eta),y(\eta))=\sum_{i=0}^{m}a_i(x(\eta))y(\eta)^i=0, \tag{3.8}$$

where $a_i(x)$ $(i=0,1,...,m)$ are polynomials of x and all relatively prime in $\mathbb{C}[x,y]$, $a_m(x)\neq 0$. Eq. (3.8) is also called the first integral to (3.7). We start our study by assuming $m=1$ in (3.8). Note that $\frac{dp}{d\eta}$ is a polynomial in x and y, and $p[x(\eta),y(\eta)]=0$ implies $\frac{dp}{d\eta}|_{(3.7)}=0$. By the Division Theorem, there exists a polynomial $H(x,y)=h(x)+g(x)y$ in $\mathbb{C}[x,y]$ such that

$$\begin{aligned}\frac{dp}{d\eta}|_{(3.7)} &= \left(\frac{\partial p}{\partial x}\frac{\partial x}{\partial \eta}+\frac{\partial p}{\partial y}\frac{\partial y}{\partial \eta}\right)|_{(3.7)}\\ &= \sum_{i=0}^{1} a_i'(x)y^{i+1}+\sum_{i=0}^{1} i a_i(x)y^{i-1}\Big((\lambda+k^2)x+\frac{1}{\beta-2k}x^3\Big)\\ &= \Big(h(x)+g(x)y\Big)\Big(\sum_{i=0}^{1} a_i(x)y^i\Big), \end{aligned} \tag{3.9}$$

where prime denotes differentiating with respect to the variable x. On equating the coefficients of y^i $(i=2,1,0)$ on both sides of (3.9), we have

$$a_1'(x)=g(x)a_1(x), \tag{3.10}$$

$$a_0'(x)=h(x)a_1(x)+g(x)a_0(x), \tag{3.11}$$

$$a_1(x)\Big((\lambda+k^2)x+\frac{1}{(\beta-2k)}x^3\Big)=h(x)a_0(x). \tag{3.12}$$

Since, $a_1(x)$ is a polynomial of x, from (3.10) we conclude that $a_1(x)$ is a constant and $g(x)=0$. For simplicity, we take $a_1(x)=1$, and balancing the degrees of $h(x)$ and $a_0(x)$ we conclude that deg $h(x)=1$, only. Now suppose that $h(x)=Ax+B$, then From (3.11), we find

$$a_0(x)=\frac{1}{2}Ax^2+Bx+D,$$

where D is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$ and $h(x)$ in (3.12) and setting all the coefficients of powers x to be zero, we obtain a system of nonlinear algebraic equations and by solving it, we obtain the following solutions:

$$A=\frac{\sqrt{2}}{\sqrt{\beta-2k}},\ B=0,\ D=\frac{\sqrt{2}}{2}(\lambda+k^2)\sqrt{\beta-2k}, \tag{3.13}$$

and

$$A=-\frac{\sqrt{2}}{\sqrt{\beta-2k}},\ B=0,\ D=-\frac{\sqrt{2}}{2}(\lambda+k^2)\sqrt{\beta-2k}. \tag{3.14}$$

Using (3.13) and (3.14) in (3.8), we obtain

$$y+\frac{\sqrt{2}}{2\sqrt{\beta-2k}}\,x^2+\frac{\sqrt{2}(\lambda+k^2)\sqrt{\beta-2k}}{2}=0,$$

and

$$y-\frac{\sqrt{2}}{2\sqrt{\beta-2k}}\,x^2-\frac{\sqrt{2}(\lambda+k^2)\sqrt{\beta-2k}}{2}=0,$$

respectively. Combining this equations with (3.7), we obtain the exact solutions of Eq. (3.6) as follows:

$$u_1(\eta) = \sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(-\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,\eta - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1),$$
$$u_2(\eta) = \sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,\eta - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1),$$

where c_1 is an arbitrary constant. Therefore, the exact solutions to (3.6) can be written as

$$u_1(x,y,t) = \sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(-\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1),$$
$$u_2(x,y,t) = \sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1).$$

Then exact solutions for system (1.1) are

$$\begin{cases} R_1 = -(\lambda+k^2)\tanh^2(-\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1), \\ Q_1 = e^{i(kx+\alpha y+\lambda t+l)}\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(-\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1). \end{cases} \tag{3.15}$$

and

$$\begin{cases} R_2 = -(\lambda+k^2)\tanh^2(\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1), \\ Q_2 = e^{i(kx+\alpha y+\lambda t+l)}\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt) - \sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1). \end{cases} \tag{3.16}$$

Now we assume that $m = 2$ in (3.8). By the Division Theorem, there exists a polynomial $H(x,y) = h(x)+g(x)y$ in $\mathbb{C}[x,y]$ such that

$$\begin{aligned} \frac{dp}{d\eta}|_{(3.7)} &= \left(\frac{\partial p}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial \eta}\right)|_{(3.7)} \\ &= \sum_{i=0}^{2} a_i'(x)y^{i+1} + \sum_{i=0}^{2} ia_i(x)y^{i-1}\Big((\lambda+k^2)x + \frac{1}{\beta-2k}x^3\Big) \\ &= \Big(h(x)+g(x)y\Big)\Big(\sum_{i=0}^{2} a_i(x)y^i\Big). \end{aligned} \tag{3.17}$$

On equating the coefficients of y^i $(i = 3,2,1,0)$ on both sides of (3.17), we have

$$a_2'(x) = g(x)a_2(x), \tag{3.18}$$

$$a_1'(x) = h(x)a_2(x) + g(x)a_1(x), \tag{3.19}$$

$$a_0'(x) = -2a_2(x)\Big((\lambda+k^2)x + \frac{1}{(\beta-2k)}x^3\Big) + h(x)a_1(x) + g(x)a_0(x), \tag{3.20}$$

$$a_1(x)\Big((\lambda+k^2)x + \frac{1}{(\beta-2k)}x^3\Big) = h(x)a_0(x). \tag{3.21}$$

Since, $a_2(x)$ is a polynomial of x, from (3.18) we conclude that $a_2(x)$ is a constant and $g(x) = 0$. For simplicity, we take $a_2(x) = 1$, and balancing the degrees of $h(x)$, $a_0(x)$ and $a_1(x)$ we conclude that deg $h(x) = 1$ or 0, therefore we have two cases:

Case 1:

Suppose that deg $h(x)=1$ and $h(x)=Ax+B$, then from (3.19) we find

$$a_1(x)=\frac{1}{2}Ax^2+Bx+D,$$

where D is an arbitrary integration constant. From (3.20) we find

$$a_0(x)=\left(\frac{A^2}{8}-\frac{1}{2(\beta-2k)}\right)x^4+\frac{AB}{2}x^3+\left(\frac{1}{2}(AD+B^2-2(\lambda+k^2))\right)x^2+BDx+E,$$

where E is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$ and $h(x)$ in (3.21) and setting all the coefficients of powers x to be zero, we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$E=\tfrac{(\lambda+k^2)^2(\beta-2k)}{2},\ B=0,\ D=(\lambda+k^2)\sqrt{2(\beta-2k)},\ A=\tfrac{2\sqrt{2}}{\sqrt{\beta-2k}}, \tag{3.22}$$

and

$$E=\tfrac{(\lambda+k^2)^2(\beta-2k)}{2},\ B=0,\ D=-(\lambda+k^2)\sqrt{2(\beta-2k)},\ A=-\tfrac{2\sqrt{2}}{\sqrt{\beta-2k}}. \tag{3.23}$$

Using (26) and (3.23) in (3.8), we obtain

$$y+\frac{1}{\sqrt{2(\beta-2k)}}x^2+\frac{(\lambda+k^2)\sqrt{\beta-2k}}{\sqrt{2}}=0,$$

and

$$y-\frac{1}{\sqrt{2(\beta-2k)}}x^2-\frac{(\lambda+k^2)\sqrt{\beta-2k}}{\sqrt{2}}=0,$$

respectively. Combining this equations with (3.7), we obtain two exact solutions to equation (3.6) which was obtained in case $m=1$, i.e.

$$u_1(\eta)=\sqrt{2k-\beta}\sqrt{k^2+\lambda}\tanh\left(-\frac{\sqrt{2}}{2}i\sqrt{k^2+\lambda}\,\eta-\sqrt{2k-\beta}\sqrt{k^2+\lambda}c_1\right),$$
$$u_2(\eta)=\sqrt{2k-\beta}\sqrt{k^2+\lambda}\tanh\left(\frac{\sqrt{2}}{2}i\sqrt{k^2+\lambda}\,\eta-\sqrt{2k-\beta}\sqrt{k^2+\lambda}c_1\right).$$

where c_1 is an arbitrary constant. Therefore, the exact solutions to (3.6) can be written as

$$u_1(x,y,t)=\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(-\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1),$$
$$u_2(x,y,t)=\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1).$$

Then the exact solutions for system (1.1) are:

$$\begin{cases} R_1=-(\lambda+k^2)\tanh^2(-\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1),\\ Q_1=e^{i(kx+\alpha y+\lambda t+l)}\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(-\tfrac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\,(x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1). \end{cases} \tag{3.24}$$

and

$$\begin{cases} R_2 = -(\lambda+k^2)\tanh^2(\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\ (x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1), \\ Q_2 = e^{i(kx+\alpha y+\lambda t+l)}\sqrt{2k-\beta}\sqrt{\lambda+k^2}\tanh(\frac{\sqrt{2}}{2}i\sqrt{\lambda+k^2}\ (x+\beta y-2kt)-\sqrt{2k-\beta}\sqrt{\lambda+k^2}c_1). \end{cases} \tag{3.25}$$

Case 2:

In this case suppose that deg $h(x) = 0$ and $h(x) = A$, then from (3.19) we find

$$a_1(x) = Ax + B,$$

where B is an arbitrary integration constant. From (3.20) we find

$$a_0(x) = \frac{-1}{2(\beta-2k)}x^4 + \left(\frac{A^2}{2} - (\lambda+k^2)\right)x^2 + ABx + D,$$

where D is an arbitrary integration constant. Substituting $a_0(x)$, $a_1(x)$ and $h(x)$ in (3.21) and setting all the coefficients of powers x to be zero, we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$A = 0,\ B = 0. \tag{3.26}$$

Using (3.26) in (3.8), we obtain

$$y^2 - (\lambda+k^2)x^2 - \frac{1}{2(\beta-2k)}x^4 = 0.$$

Combining this equations with (3.7), we obtain the exact solutions to equation (3.6) as follows:

$$u_3(\eta) = \frac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-\eta\sqrt{\lambda+k^2}\right)}}{-4e^{2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}}+e^{-2\eta\sqrt{\lambda+k^2}}},$$

$$u_4(\eta) = \frac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-\eta\sqrt{\lambda+k^2}\right)}}{1-4e^{\left(2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-2\eta\sqrt{\lambda+k^2}\right)}},$$

where c_1 is an arbitrary constant. Then the exact solutions to (3.6) can be written as:

$$u_3(x,y,t) = \frac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}}{-4e^{2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}}+e^{-2(x+\beta y-2kt)\sqrt{\lambda+k^2}}},$$

$$u_4(x,y,t) = \frac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}}{1-4e^{\left(2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-2(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}},$$

Then solutions of system (1.1) are

$$\begin{cases} R_3 = \dfrac{-32(\lambda+k^2)e^{2\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}}{\left(-4e^{2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}}+e^{-2(x+\beta y-2kt)\sqrt{\lambda+k^2}}\right)^2}, \\ Q_3 = \dfrac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}+i(kx+\alpha y+\lambda t+l)\right)}}{-4e^{2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}}+e^{-2(x+\beta y-2kt)\sqrt{\lambda+k^2}}}. \end{cases} \tag{3.27}$$

and

$$\begin{cases} R_4 = \dfrac{-32(\lambda+k^2)e^{2\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}}{\left(1-4e^{\left(2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-2(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}\right)^2}, \\ Q_4 = \dfrac{4\sqrt{2}\sqrt{\beta-2k}\sqrt{\lambda+k^2}e^{\left(\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-(x+\beta y-2kt)\sqrt{\lambda+k^2}+i(kx+\alpha y+\lambda t+l)\right)}}{1-4e^{\left(2\sqrt{2}c_1\sqrt{\beta-2k}\sqrt{\lambda+k^2}-2(x+\beta y-2kt)\sqrt{\lambda+k^2}\right)}}. \end{cases} \tag{3.28}$$

4 Exact solutions for Eq. (1.2)

In order to seek exact solutions of system (1.2), we suppose

$$Q(x,y,t) = u(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{4.1}$$

$$S(x,y,t) = v(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{4.2}$$

Where k,α and λ are constants to be determined later, l is an arbitrary constant. Substituting Eqs. (4.1) and (4.2) into system (1.2) and yields

$$\begin{cases} i(u_t+2ku_x)+u_{xx}-(\lambda+k^2)u+uR=0, \\ i(v_t+2kv_x)+v_{xx}-(\lambda+k^2)v+vR=0, \\ R_t+R_y+(u^2)_x=0, \end{cases} \tag{4.3}$$

Using the transformation

$$u=u(\eta), \quad v=v(\eta), \quad R=R(\eta), \quad \eta=x+\beta y-2kt, \tag{4.4}$$

where β is a constant, system (4.3) become the following

$$\begin{cases} u''-(\lambda+k^2)u+uR=0, \\ v''-(\lambda+k^2)v+vR=0, \\ (\beta-2k)R'+((u+v)^2)'=0, \end{cases} \tag{4.5}$$

Where prime denotes the differential with respect to η. Integrating the third segment of system (4.5) with respect to η and taking the integration constant as zero yields

$$R=\frac{-1}{\beta-2k}(u+v)^2. \tag{4.6}$$

Substituting Eq. (4.6) into the first and second segments of (4.5) yields

$$\begin{cases} u''-(\lambda+k^2)u-\frac{1}{\beta-2k}u(u+v)^2=0, \\ v''-(\lambda+k^2)v-\frac{1}{\beta-2k}v(u+v)^2=0. \end{cases} \tag{4.7}$$

We can not solve the two equations directly, but we can give a simple relations between u and v. Here we set

$$v=ru, \tag{4.8}$$

where r is an arbitrary constant. Substituting (4.8) into the system (4.7), we have

$$u''-(\lambda+k^2)u-\frac{(1+r)^2}{\beta-2k}u^3=0. \tag{4.9}$$

In the following, giving the same method to study Eq. (3.6) and denoting its solutions by $u = JE(\eta)$, we have that the solutions of system (4.7) would be

$$u = JE(\eta), \quad v = rJE(\eta). \tag{4.10}$$

Thus, solutions of system (4.3) are

$$u = JE(x+\beta y-2kt), \quad v = rJE(x+\beta y-2kt), \quad R = \frac{-(1+r)^2}{\beta-2k}(JE(x+\beta y-2kt))^2. \tag{4.11}$$

Then, solutions of system (1.2) are

$$\begin{cases} Q(x,y,t) = JE(x+\beta y-2kt)\exp[i(kx+\alpha y+\lambda t+l)], \\ S(x,y,t) = rJE(x+\beta y-2kt)\exp[i(kx+\alpha y+\lambda t+l)], \\ R(x,y,t) = \frac{-(1+r)^2}{\beta-2k}(JE(x+\beta y-2kt))^2. \end{cases} \tag{4.12}$$

where $\alpha, \beta, \lambda, l, k, r$ are constant.

5 Exact solutions for Eq. (1.3)

In order to seek exact solutions of system (1.3), we suppose

$$Q(x,y,t) = u(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{5.1}$$

$$S(x,y,t) = v(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{5.2}$$

$$N(x,y,t) = w(x,y,t)\exp[i(kx+\alpha y+\lambda t+l)], \tag{5.3}$$

Where k, α and λ are constants to be determined later, l is an arbitrary constant. Substituting Eqs. (5.1) and (5.3) into system (1.3) and yields

$$\begin{cases} i(u_t+2ku_x)+u_{xx}-(\lambda+k^2)u+uR=0, \\ i(v_t+2kv_x)+v_{xx}-(\lambda+k^2)v+vR=0, \\ i(w_t+2kw_x)+w_{xx}-(\lambda+k^2)w+wR=0, \\ R_t+R_y+(u^2)_x=0, \end{cases} \tag{5.4}$$

Using the transformation

$$u = u(\eta), \quad v = v(\eta), \quad w = w(\eta), \quad R = R(\eta), \quad \eta = x+\beta y-2kt, \tag{5.5}$$

where β is a constant, system (5.4) become the following

$$\begin{cases} u''-(\lambda+k^2)u+uR=0, \\ v''-(\lambda+k^2)v+vR=0, \\ w''-(\lambda+k^2)w+wR=0, \\ (\beta-2k)R'+((u+v+w)^2)'=0, \end{cases} \tag{5.6}$$

Where prime denotes the differential with respect to η. Integrating the third segment of system (5.6) with respect to η and taking the integration constant as zero yields

$$R = \frac{-1}{\beta - 2k}(u+v+w)^2. \tag{5.7}$$

Substituting Eq. (5.7) into other segments of (5.6) yields

$$\begin{cases} u'' - (\lambda + k^2)u - \frac{1}{\beta-2k}u(u+v+w)^2 = 0, \\ v'' - (\lambda + k^2)v - \frac{1}{\beta-2k}v(u+v+w)^2 = 0, \\ w'' - (\lambda + k^2)w - \frac{1}{\beta-2k}w(u+v+w)^2 = 0. \end{cases} \tag{5.8}$$

Of course, we can not solve system (5.8) directly, thus we should give a simple relations between u, v and w. Here we set

$$v = r_1 u, \quad w = r_2 u \tag{5.9}$$

where r_1 and r_2 are arbitrary constants. Substituting (5.9) into the system (5.8), we have

$$u'' - (\lambda + k^2)u - \frac{(1+r_1+r_2)^2}{\beta - 2k}u^3 = 0. \tag{5.10}$$

Following the same methods of the study to Eq. (3.6) and denoting the solutions of Eq. (5.10) by $u = JE(\eta)$, we have that the solutions of system (5.8) be In the following

$$u = JE(\eta), \quad v = r_1 JE(\eta), \quad w = r_2 JE(\eta). \tag{5.11}$$

Thus, solutions of system (5.4) are

$$u = JE(x+\beta y - 2kt), \quad v = r_1 JE(x+\beta y - 2kt), \quad w = r_2 JE(x+\beta y - 2kt), \tag{5.12}$$

$$R = \frac{-(1+r_1+r_2)^2}{\beta - 2k}(JE(x+\beta y - 2kt))^2.$$

Then, solutions of system (1.3) are

$$\begin{cases} Q(x,y,t) = JE(x+\beta y - 2kt)\exp[i(kx+\alpha y+\lambda t+l)], \\ S(x,y,t) = r_1 JE(x+\beta y - 2kt)\exp[i(kx+\alpha y+\lambda t+l)], \\ N(x,y,t) = r_2 JE(x+\beta y - 2kt)\exp[i(kx+\alpha y+\lambda t+l)], \\ R(x,y,t) = \frac{-(1+r_1+r_2)^2}{\beta-2k}(JE(x+\beta y - 2kt))^2. \end{cases} \tag{5.13}$$

where $\alpha, \beta, \lambda, l, k, r_1, r_2$ are constant.

6 Conclusion

We described this method for finding some exact solutions of different coupled nonlinear Maccari's systems. The solutions obtained are expressed by the hyperbolic and exponential functions. These solutions may be important for the explanation of some practical physical problems.

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